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Stochastic and Deterministic Models of Targeting, with Dynamic and Error-Prone BDA

by

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Deep precision strike is a generic military operation that depends importantly on C4/ISR system contributions. Information from the latter is realistically subject to chance influences: targets are found and correctly identified generally at rates proportional to their numbers, locations, and activities, and to the coverage of shooter-serving sensors; the events of detection are realistically random, as are the delays, results, outcomes, and follow-up of the targeting shooters. In this paper a simplified version of the above complicated process is analyzed mathematically, here as a multi-stage queuing process with imperfect service. The probabilistic outcomes can be used to anticipate the results of higher-resolution simulations; these often are far more time consuming both to set up and run.

Aspects of the above queuing situations can also be deduced via a deterministic "fluid" queuing approximation that gives an adequate and convenient representation of aspects of the state variables and various Measures of Effectiveness in the stochastic queuing model. Relying on that agreement, we have elsewhere generalized the stochastic queuing model setup to fluid models that incorporate omitted realities, such as losses from target-list tracking, and the inevitable time dependencies, non-stationarities, and adaptive behaviors that typically occur in actual military operations or vignettes. Both the stochastic and deterministic model results are informative and produce reasonable insights. Further validation steps using mathematical probability techniques as well as simulation are planned; some are in progress.

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Abstract and Summary

Deep precision strike is a generic military operation that depends importantly on C4/ISR system contributions. Information from the latter is realistically subject to chance influences: targets are found and correctly identified generally at rates proportional to their numbers, locations, and activities, and to the coverage of shooter-serving sensors; the events of detection are realistically random, as are the delays, results, outcomes, and follow-up of the targeting shooters. In this paper a simplified version of the above complicated process is analyzed mathematically, here as a multi-stage queuing process with imperfect service. The probabilistic outcomes can be used to anticipate the results of higher-resolution simulations; these often are far more time consuming both to set up and run.

Aspects of the above queuing situations can also be deduced via a deterministic "fluid" queuing approximation that gives an adequate and convenient representation of aspects of the state variables and various Measures of Effectiveness in the stochastic queuing model. Relying on that agreement, we have elsewhere generalized the stochastic queuing model setup to fluid models that incorporate omitted realities, such as losses from target-list tracking, and the inevitable time dependencies, non-stationarities, and adaptive behaviors that typically occur in actual military operations or vignettes. Both the stochastic and deterministic model results are informative and produce reasonable insights. Further validation steps using mathematical probability techniques as well as simulation are planned; some are in progress.

- **It is ironic, but of interest and potential value, that strong abstract correspondence exists between the *deep, precision strike* situation described and for which our models have been formulated, and certain *disaster relief* scenarios. In these, the deep-strike targets are identified as disaster victims (human, or infrastructure), the C4/ISR sensor assets are discovery-medical diagnosis and triage systems, and the deep-strike weaponry ("shooters") is replaced by suppliers of medical service. In both cases time sensitivity and uncertainty exacerbate the decision problems. Force composition and structure questions translate into very similar questions and issues for these two important topics of modern military concern. This is an argument for the cost-effectiveness of pursuing an abstract *model type* that has a range of applications and provides broad insights.**

0. Overview

Battle damage assessment (BDA) is an aspect of "hard" battlespace information war (IW)/information operations (IO) that promises to add to the efficiency of combat engagements. In spite of the precision of modern weaponry and sensor/communication system, shots fired at targets will occasionally miss (or cause only partial damage). Consequently a sequence of several shots may be directed at a particular (hostile) target to increase the probability of kill. This paper examines the efficacy of a shooting strategy that depends upon information: that of shoot-look-shoot.

In the shoot-look-shoot tactic the targeter (Blue) fires once at an acquired target (Red). He then "looks" at it and classifies it as alive or dead: if the target is classified as alive, he shoots at it only once more. An acquired target is never shot at more than twice on a particular occasion. A target that has been shot at twice must be reacquired to be classified as being alive or dead; that is, a (hostile) target that survives two shots must be reacquired to receive more shots; in the meantime it can itself launch weapons, or advance to pursue an advantage.

An effective BDA capacity can greatly reduce the opponent's options and effectiveness by increasing the chance that a targeting mission is successful *and* that this fact is known to the targeter. On the other hand, seriously error-prone BDA tends to clog target lists with unprofitable already-dead "targets" that vastly hamper the shooter's response time, hence kill rate, and wastefully inflate the expenditure of missile inventory.

Section 1 describes and presents results for a stochastic queuing network model of the situation described. The queuing network model allows closed-form calculation of *long-run* distributional results that are easily turned into numbers and graphs without the need for Monte Carlo simulation. It is almost always difficult to obtain mathematically neat closed-form *time-dependent* results for such a queuing model; simulation or numerical calculations are required. Section 2 presents a deterministic or expected-value approximation to finite server queues of the type above. Sections 3 and 4 present deterministic approximations to the network queuing model of Section 1; the agreement with the mean values of the stochastic model tends to be very satisfactory, but no information on state fluctuations or risk is available from such models.

Papers that discuss similar problems and contain further references are Almeida, Gaver, and Jacobs (1995), and Gaver and Jacobs (1987); see also Evans (1996), Aviv and Kress (to appear) and Manor and Kress (to appear).

1. An Aggregated Queuing Model of Defensive Targeting when Service Success is Assessed with Error, and the Shooting Protocol is Shoot-Look-Shoot

1.1 The Model

Suppose attackers that are targets for a defensive force appear in region $\mathcal{R}$ at a *constant* Poisson rate λ. The time until an unacquired target that is not itself firing is detected by a surveillance system is distributed exponentially with mean $1/\xi$. A live target that is detected is classified as live and put on the shooter servers' targeting list with probability R_{aa}. With probability $(1 - R_{aa})$, it is misclassified as dead and returns to the unacquired state. A dead target that has not yet been classified as dead is classified as dead when it is acquired with probability R_{dd} and is removed from the system; with probability $(1 - R_{dd})$ it is classified as live, and is erroneously and wastefully put on the targeting list.

The times between shots by a live Red potential target, such as a TEL (e.g. SCUD or anti-air missile launcher) are independent identically distributed exponential (Markovian) with mean $1/\alpha$. An unacquired firing target is detected and put on the shooter's targeting list with probability p_A after it fires.

A detected target that has been classified as targetable (perhaps inappropriately because dead) is viewed as queued and awaiting attention of one of s ($s = 1, 2, \ldots$) shooters/"servers"; these can be thought of as missile launchers. Service times for a shooter can be viewed as realizations of a random variable that includes, implicitly, time for the target waiting in the detected queue (residing on the *target list)*, conveyed by C4ISR, to be converted to tracking-firing information; it also includes time of flight in this model.

We assume a shooter server uses a *shoot-look-shoot* protocol. A shot kills a target with probability p_K. A (possibly erroneous) battle damage assessment occurs immediately after the first shot. If the first shot kills the target then, with probability C_{dd}, the target is correctly classified as dead and is appropriately ignored from then on; with probability $(1 - C_{dd})$ the target is incorrectly classified as live and it is shot at once again. If the first shot misses the target, then with the probability C_{aa} the target is classified as live and the target is shot at a second time; with probability $(1 - C_{aa})$ the target is misclassified as dead and returns to an unacquired state. No battle damage assessment occurs after the second shot; the shooter immediately moves to the next enqueued targetable unit. Once a dead target is classified as dead it is taken out of the system.

1.2 Number of Times a Target is Shot At

In this section we obtain expressions for the expected number of times a target is shot at with variations in the way the implied question is phrased.

1.2.1 Number of times a target that starts as unacquired is shot at while it is alive: S_{AA}

$$E[S_{AA}] = \underbrace{1p_K}_{\substack{\text{1st shot}\\ \text{kills the}\\ \text{target}}} + \underbrace{2(1-p_K)C_{aa}p_K}_{\substack{\text{1st shot misses the}\\ \text{target; the target is}\\ \text{correctly classified;}\\ \text{the 2nd shot kills}\\ \text{the target}}} + (1-p_K)C_{aa}(1-p_K)[2+E[S_{AA}]]$$

$$+(1-p_K)(1-C_{aa})[1+E[S_{AA}]].$$

Solving,

$$E[S_{AA}] = \frac{p_K + 2(1-p_K)C_{aa} + (1-p_K)(1-C_{aa})}{1-(1-p_K)C_{aa}(1-p_K)-(1-p_K)(1-C_{aa})} = \frac{1}{p_K}. \qquad (1.2.1)$$

Note that $E[S_{AA}]$ depends only on p_K, despite the uncertainties of perception ($C_{aa} < 1$); also, the result does not depend on C_{dd} or $C_{da} = 1 - C_{da}$. The BDA process has no influence on this particular measure.

1.2.2 Number of times a dead target that starts as unacquired is shot at until it is *classified as dead*: S_D

$$E[S_D] = \underbrace{R_{dd} \times 0}_{\substack{\text{prob the}\\ \text{sensor}\\ \text{correctly}\\ \text{classifies}\\ \text{the dead}\\ \text{target}}} + \underbrace{(1-R_{dd})}_{\substack{\text{prob the}\\ \text{sensor}\\ \text{incorrectly}\\ \text{classifies}\\ \text{the dead}\\ \text{target}}} \Bigg[\underbrace{1C_{dd}}_{\substack{\text{shooter}\\ \text{takes one}\\ \text{shot and}\\ \text{correctly}\\ \text{classifies}\\ \text{target}}} + \underbrace{(1-C_{dd})}_{\substack{\text{shooter}\\ \text{misclassifies}\\ \text{target after}\\ \text{1st shot}}} [2+E[S_D]] \Bigg].$$

Solving

$$E[S_D] = \frac{(1-R_{dd})[1+(1-C_{dd})]}{1-(1-R_{dd})(1-C_{dd})}. \qquad (1.2.2)$$

Note that this does not depend on the true kill probability, p_K, neither does it depend on C_{aa}.

1.2.3 Number of times a live target that starts as unacquired is shot at until it is killed and classified as killed: S_{AD}

$$E[S_{AD}] = \underbrace{p_K C_{dd}}_{\substack{\text{prob target}\\ \text{is killed on}\\ \text{1st shot and}\\ \text{classified}\\ \text{as dead}}} + \Big[\underbrace{(1-p_K)C_{aa}p_K}_{\substack{\text{prob target is killed}\\ \text{on 2nd shot}}} + \underbrace{p_K(1-C_{dd})}_{\substack{\text{prob killed}\\ \text{on 1st shot and}\\ \text{misclassified so}\\ \text{shot at 2nd time}}} \Big][2+E[S_D]]$$

$$+ \underbrace{(1-p_K)(1-C_{aa})}_{\substack{\text{prob 1st shot}\\ \text{misses and target}\\ \text{misclassified}\\ \text{as dead}}}[1+E[S_{AD}]] + \underbrace{(1-p_K)^2 C_{aa}}_{\substack{\text{prob 1st shot}\\ \text{misses, target}\\ \text{correctly classified}\\ \text{as live and 2nd}\\ \text{shot also misses}}} [2+E[S_{AD}]]$$

Solving

$$E[S_{AD}] =$$

$$\frac{p_K C_{dd} + [(1-p_K)C_{aa}p_K + p_K(1-C_{dd})][2+E[S_D]] + (1-p_K)(1-C_{aa}) + 2(1-p_K)^2 C_{aa}}{1-\left[(1-p_K)(1-C_{aa}) + (1-p_K)^2 C_{aa}\right]}$$

$$= \frac{1}{p_K} + E[S_D] + \frac{p_K[R_{dd} - C_{dd}]}{\left[1-\left[(1-p_K)(1-C_{aa}) + (1-p_K)^2 C_{aa}\right]\right]\left[1-(1-R_{dd})(1-C_{dd})\right]}$$

$$= \frac{1}{p_K} + E[S_D] + \frac{p_K[R_{dd} - C_{dd}]}{[1-(1-p_K)[1-p_K C_{aa}]]\,[1-(1-R_{dd})(1-C_{dd})]} \qquad (1.2.3)$$

$$= \frac{1}{p_K} + \frac{[(1-C_{dd})[1+(1-R_{dd})] + (1-p_K)C_{aa}(1-R_{dd})[1+(1-C_{dd})]]}{[1+(1-p_K)C_{aa}]\,[1-(1-R_{dd})(1-C_{dd})]}.$$

Note that changes in the values of R_{dd} and C_{dd} most strongly influence $E[S_{AD}]$ through $E[S_D]$. If $C_{dd} = R_{dd}$, then $E[S_{AD}]$ is independent of C_{aa}. It is clear that the capability to correctly identify dead targets as dead is of great importance to minimize wasted shots, and (1.2.3) quantifies this dramatically: for small $C_{dd} = R_{dd}$, the above reduces to $E[S_{AD}] \cong 1/p_K + 1/C_{dd}$.

1.2.4 Number of times a live target that starts as unacquired is shot at while it is dead: S_{DD}

$$\begin{aligned} E[S_{DD}] &= E[S_{AD}] - E[S_{AA}] \\ &\cong 1/C_{dd} \qquad \text{if } C_{dd} \text{ is small.} \end{aligned} \tag{1.2.4}$$

1.3. Number of Times a Target Passes Through the Surveillance System

In this section we obtain expressions for the expected number of times a target passes through the surveillance system.

1.3.1 Number of times a dead target that starts as unacquired passes through the surveillance server until it is classified as dead: L_D

$$E[L_D] = \underbrace{1R_{dd}}_{\substack{\text{sensor}\\ \text{correctly}\\ \text{classifies}\\ \text{dead}\\ \text{target}}} + 1(1-R_{dd})\cdot \underbrace{C_{dd}}_{\substack{\text{shooter}\\ \text{correctly}\\ \text{classifies}\\ \text{target after}\\ \text{1st shot}}} + (1-R_{dd})(1-C_{dd})[1+E[L_D]]$$

Solving

$$\begin{aligned} E[L_D] &= \frac{1}{1-(1-R_{dd})(1-C_{dd})} \\ &\cong \frac{1}{(R_{dd}+C_{dd})} \end{aligned} \tag{1.3.1}$$

if R_{dd} and C_{dd} are small. This indicates the extra load imposed by futilely processing dead targets.

1.3.2 Number of times a live target that starts as unacquired passes through the surveillance server until it is killed and classified as dead: L_{AD}

Let L_{AD} be the number of times an unacquired live target passes through the surveillance server until it is killed and classified as dead.

$$E[L_{AD}] = \underbrace{\frac{\xi}{\xi+\alpha}}_{\substack{\text{prob live}\\ \text{target}\\ \text{detected}\\ \text{by sensor}}} \underbrace{(1-R_{aa})}_{\substack{\text{prob live}\\ \text{target}\\ \text{misclassified}\\ \text{by sensor}}} [1+E[L_{AD}]]$$

$$\frac{\xi}{\xi+\alpha}\Big\{R_{aa}\big[1+ \underbrace{(p_K C_{dd}}_{\substack{\text{prob target}\\ \text{killed on 1st shot}\\ \text{and correctly}\\ \text{classified}}} \times 0) + p_K(1-C_{dd})E[L_D]\big]$$

$$+R_{aa}\big[1+ \underbrace{(1-p_K)}_{\substack{\text{prob target}\\ \text{not killed}\\ \text{on 1st shot}}} \underbrace{(1-C_{aa})}_{\substack{\text{prob target}\\ \text{incorrectly}\\ \text{classified}}} E[L_{AD}]$$

$$+(1-p_K)C_{aa}p_K E[L_D] + (1-p_K)C_{aa}(1-p_K)E[L_{AD}]\big]\Big\}$$

$$+ \underbrace{\frac{\alpha}{\alpha+\xi}}_{\substack{\text{prob live target}\\ \text{is detected}\\ \text{because it shoots}}} \underbrace{(1-p_A)}_{\substack{\text{prob shooting}\\ \text{target is not put}\\ \text{on targeting list}}} [1+E[L_{AD}]]$$

$$+\frac{\alpha}{\alpha+\xi}p_A\Big\{1+0p_K C_{dd} + p_K(1-C_{dd})E[L_D] + (1-p_K)\ (1-C_{aa})E[L_{AD}]$$

$$+(1-p_K)C_{aa}p_K E[L_D] + (1-p_K)C_{aa}(1-p_K)E[L_{AD}]\Big\}$$

Solving,

$E[L_{AD}] =$

$$\frac{1+\left(\frac{\xi}{\xi+\alpha}R_{aa} + \frac{\alpha}{\xi+\alpha}p_A\right)\left[p_K(1-C_{dd})E[L_D] + (1-p_K)C_{aa}p_K E[L_D]\right]}{1-\left\{\left[\frac{\xi}{\xi+\alpha}(1-R_{aa}) + \frac{\alpha}{\xi+\alpha}(1-p_A)\right] + \left[\frac{\xi}{\xi+\alpha}R_{aa} + \frac{\alpha}{\xi+\alpha}p_A\right]\left[(1-p_K)(1-C_{aa}) + (1-p_K)^2 C_{aa}\right]\right\}} \quad (1.3.2)$$

$$= \frac{1+\left(\frac{\xi}{\xi+\alpha}R_{aa} + \frac{\alpha}{\xi+\alpha}p_A\right)E[L_D]p_K\left[(1-C_{dd}) + (1-p_K)C_{aa}\right]}{\left[\frac{\xi}{\xi+\alpha}R_{aa} + \frac{\alpha}{\xi+\alpha}p_A\right]p_K\left[1+C_{aa}(1-p_K)\right]}$$

Small changes in R_{aa} and p_A, when they are small, can greatly affect $E[L_{AD}]$. Small changes in C_{dd} and R_{dd} can also greatly affect $E[L_{AD}]$ through $E[L_D]$.

1.3.3 Number of times a live target is acquired before it is killed: L_{AA}

$$E[L_{AA}] = 1 + \frac{\xi}{\xi+\alpha}\left\{(1-R_{aa})E[L_{AA}] + R_{aa}\left[(1-p_K)(1-C_{aa}) + (1-p_K)^2 C_{aa}\right]E[L_{AA}]\right\}$$
$$+\frac{\alpha}{\xi+\alpha}\left\{(1-p_A)E[L_{AA}] + p_A\left[(1-p_K)(1-C_{aa}) + (1-p_K)^2 C_{aa}\right]E[L_{AA}]\right\}$$

Solving

$$E[L_{AA}] =$$
$$\frac{1}{\left[1-\left\{\frac{\xi}{\xi+\alpha}(1-R_{aa}) + \frac{\alpha}{\xi+\alpha}(1-p_A) + \left[\frac{\xi}{\xi+\alpha}R_{aa} + \frac{\alpha}{\xi+\alpha}p_A\right]\left[(1-p_K)(1-C_{aa}) + (1-p_K)^2 C_{aa}\right]\right\}\right]}$$
$$= \frac{1}{\left[1-\left\{\left[\frac{\xi}{\xi+\alpha}(1-R_{aa}) + \frac{\alpha}{\xi+\alpha}(1-p_A)\right] + \left[\frac{\xi}{\xi+\alpha}R_{aa} + \frac{\alpha}{\xi+\alpha}p_A\right](1-p_K)[1-p_K C_{aa}]\right\}\right]} \quad (1.3.3)$$
$$= \frac{1}{\left[\frac{\xi}{\xi+\alpha}R_{aa} + \frac{\alpha}{\xi+\alpha}p_A\right]p_K[1+C_{aa}(1-p_K)]}$$

Note that small changes in R_{aa} and p_A when they are small can result in non-linearly large changes in $E[L_{AA}]$.

We now discuss the queuing model.

1.4 Mathematical Details of the Queuing Model

Important operationally relevant questions about the system can be addressed in terms of a queuing model. The targets are customers. They are either unacquired or queued and awaiting attention of one of s $(s = 1, 2, \ldots)$ shooters/"servers".

1.4.1 The shooting server

We will say a target is of type (a_S, b_S) if it requires a_S shots while it is alive and an additional b_S shots to classify the dead target as dead. The type of each target is independent of the types of other targets. The expected number of shots required by a target that has arrived to the region is $E[S_{AD}]$. The total arrival rate of targets to the shooter service system including those that are retargeted is $\lambda E[S_{AD}]$.

Assume the target list queue for the shooters evolves as follows (cf. Kelly [1979]):

a) Each customer (target) requires an amount of service which is a random variable exponentially distributed with unit mean.

b) A total service (shooting) effort is supplied at the rate

$$\varphi(n) = \mu \min(s, n)$$

when there are n targets waiting or being served.

c) A proportion $\gamma(\ell, n)$ of this effort is directed to the customer (target) in position ℓ in the queue ($\ell = 1, 2, \ldots, n$) where

$$\gamma(\ell,n)=\begin{cases}\frac{1}{n} & \ell=1,2,\ldots,n, \quad n=1,2,\ldots,s\\ \frac{1}{s} & \ell=1,2,\ldots,s, \quad n=s+1,s+2,\ldots\\ 0 & \text{otherwise}\end{cases} \tag{1.4.1}$$

d) When a customer arrives at the queue he moves into position ℓ with probability

$$\gamma(\ell,n+1)=\begin{cases}1 & \ell=n+1\\ 0 & \text{otherwise}\end{cases} \tag{1.4.2}$$

when there are n targets waiting or being served.

The shooter service system behaves as an M/M/s queue with mean service time $1/\mu$.

Let $X_S(t)$ be the number of targets waiting for service or being served by the shooter service system at time t. Corollary 3.4 of Kelly [1979] implies that if $\lambda_S \equiv \lambda E[S_{AD}] < \mu s$, then a limiting distribution exists.

$$\lim_{t \to \infty} P\{X_S(t) = n\} = \pi_S(n)$$

with

$$\pi_S(n) = \begin{cases} \left(\dfrac{\lambda_S}{\mu}\right)^n \dfrac{1}{n!}\ \pi_S(0) & \text{for } n = 0, 1, \ldots, s \\[2ex] \left(\dfrac{\lambda_S}{\mu}\right)^s \dfrac{1}{s!}\left(\dfrac{\lambda_S}{s\mu}\right)^{n-s} \pi_S(0) & \text{for } n = s+1, s+2, \ldots \end{cases} \tag{1.4.3}$$

with

$$\pi_S(0) = \left[\sum_{n=0}^{s-1}\left(\frac{\lambda_S}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda_S}{s\mu}\right)^s \frac{1}{s!}\ \frac{1}{1-(\lambda_S/s\mu)}\right]^{-1}.$$

If $\lambda_S > s\mu$ then the effective arrival rate of targets is at least as large as the maximum service rate and $\lim_{t \to \infty} P\{X_S(t) = n\} = 0$ for $n = 0, 1, \ldots$. The servers are saturated and the population of unserved targets increases linearly beyond all bounds. Henceforth, assume $\lambda_S < \mu s$.

The long-run mean number of targets waiting for shooter service or being served is

$$E[X_S(\infty)] = \frac{\lambda_S}{\mu} + \pi_S(0)\left(\frac{\lambda_S}{\mu}\right)^s \frac{1}{s!}\ \frac{\lambda_S/\mu s}{[1-(\lambda_S/\mu s)]^2}. \tag{1.4.4}$$

The long-run mean queue length at the shooter is

$$E[Q_S] = \pi_S(0)\frac{1}{s!}\left(\frac{\lambda_S}{\mu}\right)^s \frac{\lambda_S/\mu s}{\left[1-(\lambda_S/\mu s)\right]^2}. \tag{1.4.5}$$

Both of these expressions reveal the substantial nonlinearity of shooter backlog, hence delay: if arrival rate of targets, λ, were to increase, backlog skyrockets; but a similar and synergistic effect occurs if $E[S_{AD}]$ is high because of incorrect classification. The model quantifies the possibly substantial effect of improving classification capability on overall targeting performance and can be used to study the tradeoff between good classification and traffic handling capability.

It follows from Theorem 3.1 of Kelly [1979] that the long-run mean number of live targets waiting for or receiving service by the shooter is

$$E[X_S(\infty)]\ \frac{E[S_{AA}]}{E[S_{AD}]}. \tag{1.4.6}$$

The mean number of shooter/servers that are busy is

$$E[S] = \pi_S(0)\left[\sum_{k=0}^{s-1} k\frac{(\lambda/\mu)^k}{k!} + \frac{s}{s!}\left(\frac{\lambda_S}{\mu}\right)^s \frac{1}{1-\frac{\lambda_S}{s\mu}}\right]. \tag{1.4.7}$$

The mean number of shooter/servers that are serving live targets is

$$E[S_A] = E[S]\frac{E[S_{AA}]}{E[S_{AD}]}. \tag{1.4.8}$$

The long-run mean rate at which live targets are killed is

$$\rho_K = E[S_A]\mu p_K.$$

We will model the surveillance system similarly but as behaving as an infinite server queue with mean service time $1/(\xi + \alpha)$, where α is the rate at which targetable opponents reveal themselves by taking offensive action, e.g. shooting SCUDs.

The long-run mean number of undetected targets (both live and dead) is

$$E[X_L(\infty)] = \lambda E[L_{AD}]\ \frac{1}{\xi+\alpha}. \tag{1.4.9}$$

The long-run mean number of live undetected targets is

$$\lambda E[L_{AD}]\frac{1}{\xi+\alpha}\frac{E[L_{AA}]}{E[L_{AD}]} = \lambda\frac{1}{\xi+\alpha}E[L_{AA}] \tag{1.4.10}$$

From Little's formula, the mean time it takes to kill a target ***and classify it as dead*** is

$$W = \frac{1}{\lambda}\left[E[X_L(\infty)] + E[X_S(\infty)]\right]. \tag{1.4.11}$$

The mean time it takes to kill a target is

$$W_A = E[L_{AA}]\frac{1}{\xi+\alpha} + \frac{E[S_{AA}]}{E[S_{AD}]}\frac{1}{\lambda}E[X_S(\infty)]$$

The mean number of offensive shots (SCUDs launched) by a Red target is αW_A. All of the above expressions can easily be numerically tabulated; see below.

Numerical Examples

In the numerical examples, the arrival rate of targets to the area is 15/hr; the rate of target detection by the sensors is $\xi = (1/2)$/hr; the rate of firing by a Red $\alpha = (1/2)$/hr; the service rate by a Blue server = 3 per hour; there are 20 Blue servers; the $p_K = 0.5$. This is an entirely hypothetical set of numbers and is offered only as a very roughly plausible illustration.

Figure 1.1 presents the average time to kill a Red as a function of p_A, the probability that a firing Red is put on the targeting list. Increasing p_A from 0.1 to 0.8 reduced the average time to kill a target from over 2 hours to about 1 hour.

The average time to classify a dead target as dead is about an hour. The other classification probabilities are $R_{aa} = 0.5$, $R_{dd} = 0.7$, $C_{aa} = 0.5$, $C_{dd} = 0.5$.

Figure 1.2 presents the average rate of Red shots per hour as a function of p_A. Increasing p_A from 0.1 to 0.5 reduces the Red shots per hour from about 45 to 30; further increases in p_A are less influential *unless*, say, shooting rate and/or kill probability are increased.

Figure 1.3 displays the mean number of shots fired by a Red target as a function of p_A for 2 different values of sensor acquisition rate, ξ, one "low" $\xi = 0.5$/hr and one "high" $\xi = 2$/hr. Note that if the sensor acquisition rate is high, then the value of p_A has little effect.

Figure 1.4 displays the mean number of Blue shots to kill a Red target and the mean number of Blue shots to kill a Red target and classify it as dead as a function of C_{dd}. Since $p_K = 0.5$, the mean number of shots to actually kill a Red target is 2. However, the mean number of *additional* Blue shots expended until a dead Red target is *classified as dead* can be close to 2 for small C_{dd} (it could approach ∞ if R_{dd} were also small) but is negligible for $C_{dd} \sim 1$. Ability to classify well is seen to be extremely influential on shooter system efficiency.

Figure 1.5 displays the traffic intensity at the shooting service system as C_{dd} varies. A traffic intensity larger than 1 means that the service system is unstable and won't be able to handle the work load presented to it. With other parameters fixed as shown, the value of C_{dd} must be close to 0.3 or greater in order for the queue to be stable, i.e. not to eventually grow beyond bounds. Even if $C_{dd} = 0.3$ the mean number of targets (both live and dead) waiting or being served at the service system will be unacceptably high; the queue, and delay, can be brought down quickly and substantially by increasing C_{dd}. This step also cuts into Red effectiveness.

Figure 1.6 displays the limiting distribution of the number of targets waiting for or being served by the shooter-servers (1.4.3). The model parameters are λ = 15, $\xi = 2$, $\alpha = 0.5$, $R_{aa} = 0.5$, $R_{dd} = 0.6$, $C_{aa} = 0.5$, $p_A = 0.5$, $p_K = 0.5$, $\mu = 5$, $s = 10$. The upper graph displays $\pi_S(n)$, $n = 0, 1, \ldots$ where $C_{dd} = 0.3$. The lower graph displays $\pi_S(n)$, $n = 0, 1, \ldots$ where $C_{dd} = 0.8$. Table 1.1 displays the mean and variance of the number of targets waiting for or being served by the shooter-servers.

Table 1.1
Moments for Limiting Distribution of the Number of Targets Waiting or Being Served
$\lambda = 15$, $\xi = 2$, $\alpha = 0.5$, $R_{aa} = 0.5$, $R_{dd} = 0.6$, $C_{aa} = 0.5$, $p_A = 0.5$, $p_K = 0.5$, $\mu = 5$, $s = 10$

C_{dd}	Mean	Variance
0.1	—	—
0.2	—	—
0.3	62.3	3543.0
0.4	16.9	132.3
0.5	11.8	43.9
0.6	9.6	23.3
0.7	7.4	15.4
0.8	7.6	11.5
0.9	7.0	9.3
1.0	6.4	7.9

— Queue is saturated

The poorer the ability to classify a dead target as dead, (lower C_{dd}), the greater the variability in the number of targets waiting or being served by the shooter servers. The graph of the limiting distribution with $C_{dd} = 0.3$ indicates that the distribution has a very long and heavy right hand tail; there is a sizable probability that more than 100 targets are waiting for service or being served; the heavy tail is reflected in the variance of the distribution which is 3543, compared

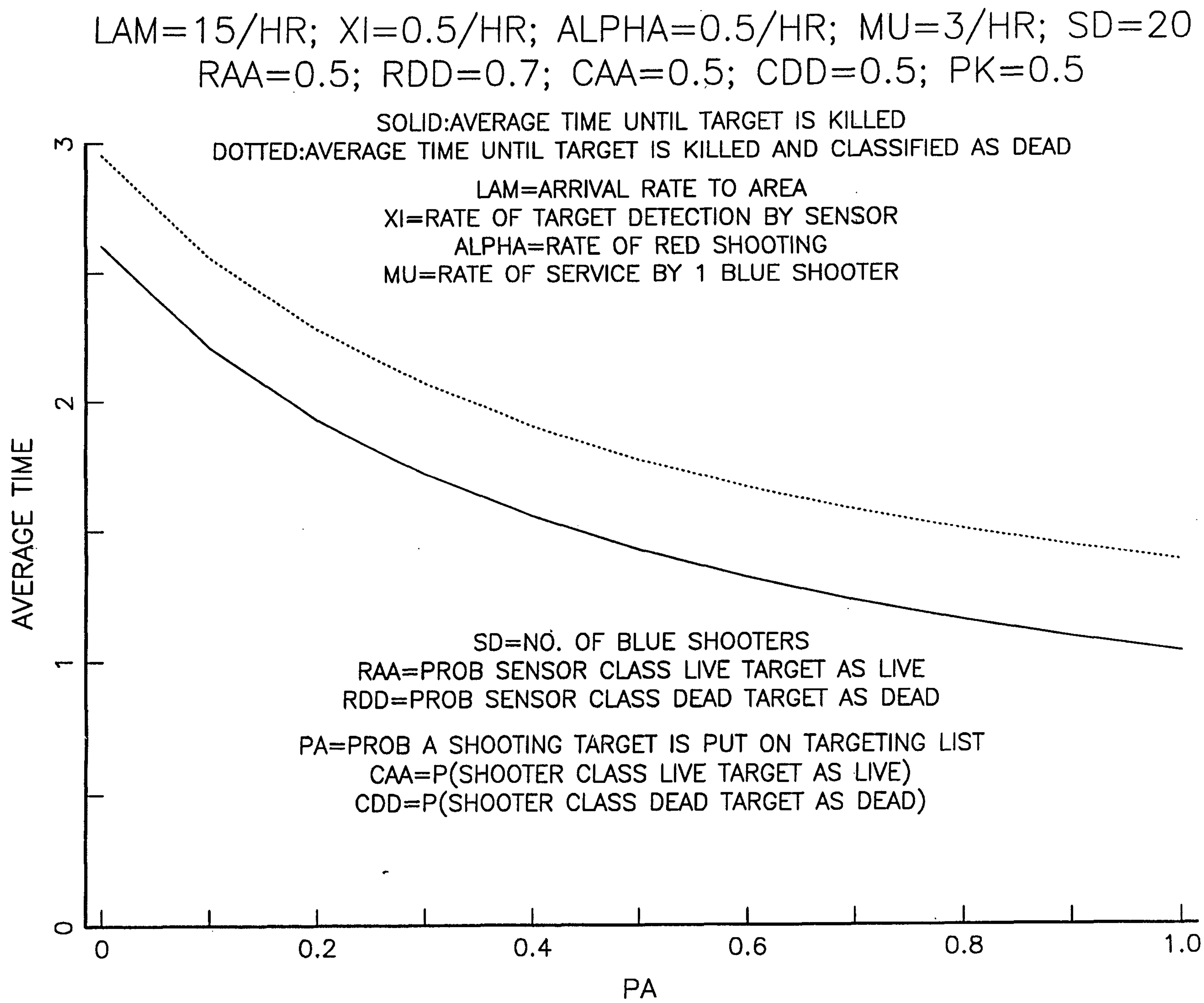

LAM=15/HR; XI=0.5/HR; ALPHA=0.5/HR; MU=3/HR; SD=20
RAA=0.5; RDD=0.7; CAA=0.5; CDD=0.5; PK=0.5
SOLID:AVERAGE TIME UNTIL TARGET IS KILLED
DOTTED:AVERAGE TIME UNTIL TARGET IS KILLED AND CLASSIFIED AS DEAD
LAM=ARRIVAL RATE TO AREA
XI=RATE OF TARGET DETECTION BY SENSOR
ALPHA=RATE OF RED SHOOTING
MU=RATE OF SERVICE BY 1 BLUE SHOOTER
SD=NO. OF BLUE SHOOTERS
RAA=PROB SENSOR CLASS LIVE TARGET AS LIVE
RDD=PROB SENSOR CLASS DEAD TARGET AS DEAD
PA=PROB A SHOOTING TARGET IS PUT ON TARGETING LIST
CAA=P(SHOOTER CLASS LIVE TARGET AS LIVE)
CDD=P(SHOOTER CLASS DEAD TARGET AS DEAD)
AVERAGE TIME
3
2
1
0
0
0.2
0.4
0.6
0.8
1.0
PA

Figure 1.1

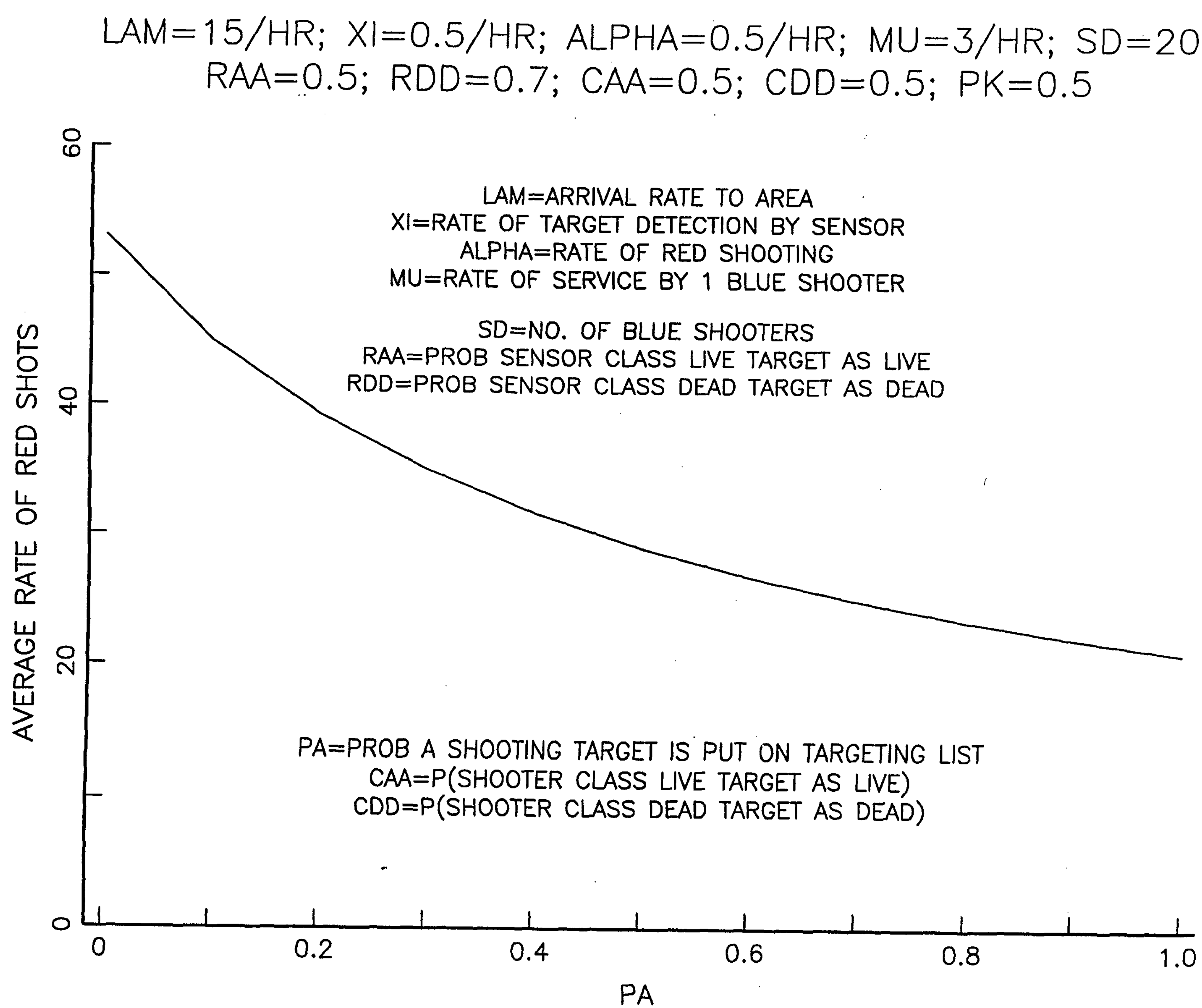

LAM=15/HR; XI=0.5/HR; ALPHA=0.5/HR; MU=3/HR; SD=20
RAA=0.5; RDD=0.7; CAA=0.5; CDD=0.5; PK=0.5
LAM=ARRIVAL RATE TO AREA
XI=RATE OF TARGET DETECTION BY SENSOR
ALPHA=RATE OF RED SHOOTING
MU=RATE OF SERVICE BY 1 BLUE SHOOTER
SD=NO. OF BLUE SHOOTERS
RAA=PROB SENSOR CLASS LIVE TARGET AS LIVE
RDD=PROB SENSOR CLASS DEAD TARGET AS DEAD
PA=PROB A SHOOTING TARGET IS PUT ON TARGETING LIST
CAA=P(SHOOTER CLASS LIVE TARGET AS LIVE)
CDD=P(SHOOTER CLASS DEAD TARGET AS DEAD)
AVERAGE RATE OF RED SHOTS
0
20
40
60
0
0.2
0.4
0.6
0.8
1.0
PA

Figure 1.2

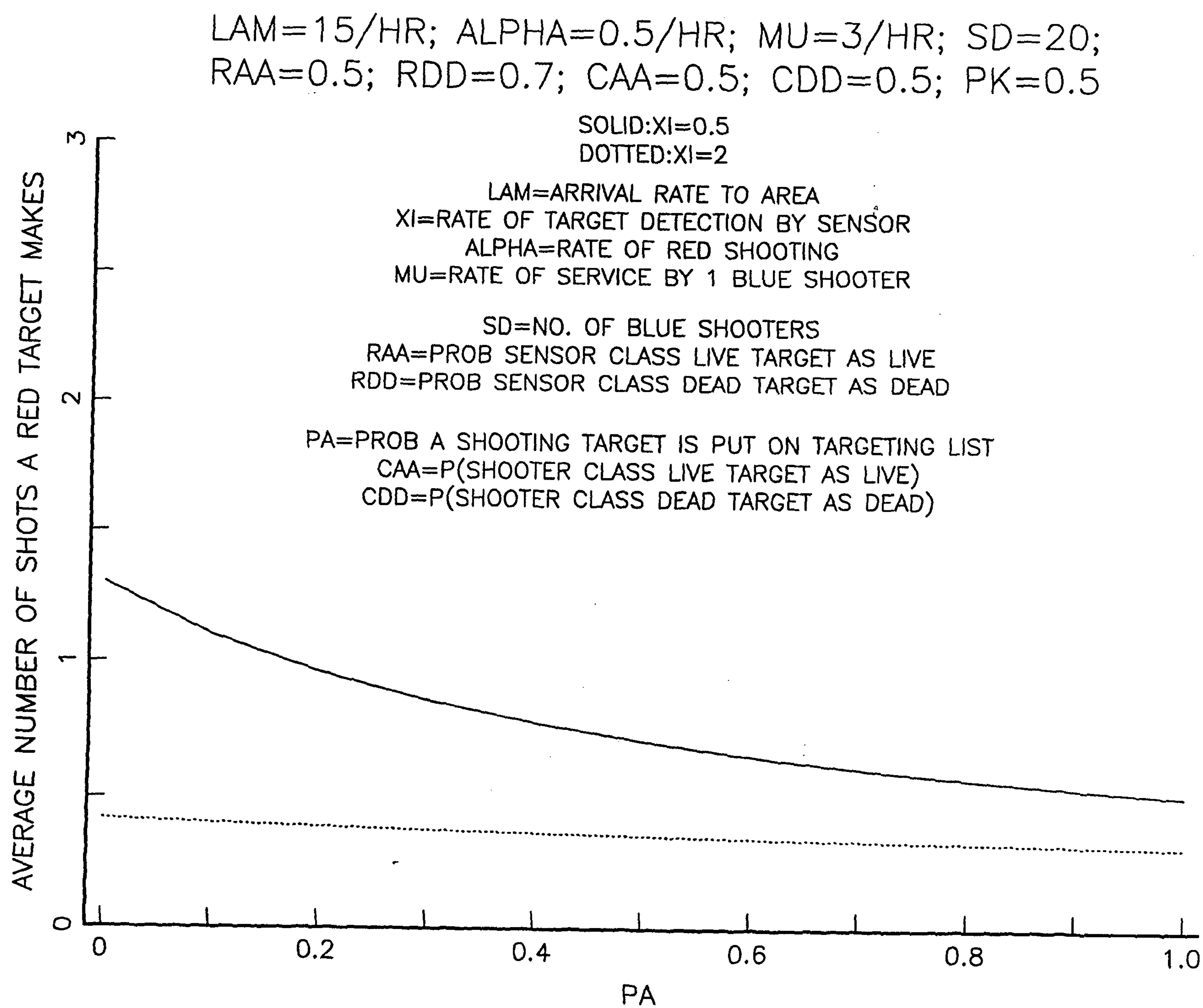

LAM=15/HR; ALPHA=0.5/HR; MU=3/HR; SD=20;
RAA=0.5; RDD=0.7; CAA=0.5; CDD=0.5; PK=0.5
SOLID:XI=0.5
DOTTED:XI=2
LAM=ARRIVAL RATE TO AREA
XI=RATE OF TARGET DETECTION BY SENSOR
ALPHA=RATE OF RED SHOOTING
MU=RATE OF SERVICE BY 1 BLUE SHOOTER
SD=NO. OF BLUE SHOOTERS
RAA=PROB SENSOR CLASS LIVE TARGET AS LIVE
RDD=PROB SENSOR CLASS DEAD TARGET AS DEAD
PA=PROB A SHOOTING TARGET IS PUT ON TARGETING LIST
CAA=P(SHOOTER CLASS LIVE TARGET AS LIVE)
CDD=P(SHOOTER CLASS DEAD TARGET AS DEAD)
AVERAGE NUMBER OF SHOTS A RED TARGET MAKES
0
1
2
3
0
0.2
0.4
0.6
0.8
1.0
PA

Figure 1.3

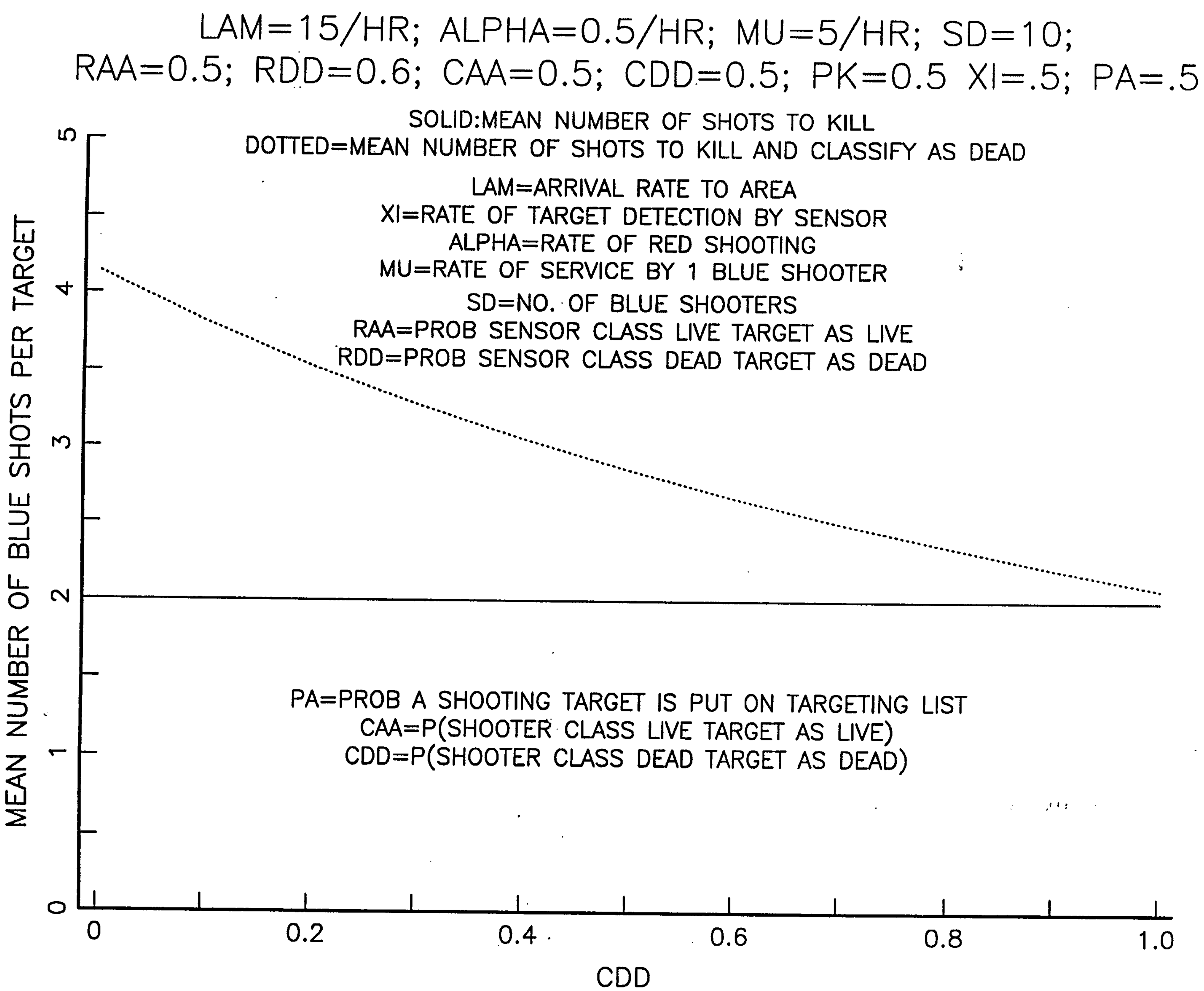

LAM=15/HR; ALPHA=0.5/HR; MU=5/HR; SD=10;
RAA=0.5; RDD=0.6; CAA=0.5; CDD=0.5; PK=0.5 XI=.5; PA=.5
SOLID:MEAN NUMBER OF SHOTS TO KILL
DOTTED=MEAN NUMBER OF SHOTS TO KILL AND CLASSIFY AS DEAD
LAM=ARRIVAL RATE TO AREA
XI=RATE OF TARGET DETECTION BY SENSOR
ALPHA=RATE OF RED SHOOTING
MU=RATE OF SERVICE BY 1 BLUE SHOOTER
SD=NO. OF BLUE SHOOTERS
RAA=PROB SENSOR CLASS LIVE TARGET AS LIVE
RDD=PROB SENSOR CLASS DEAD TARGET AS DEAD
PA=PROB A SHOOTING TARGET IS PUT ON TARGETING LIST
CAA=P(SHOOTER CLASS LIVE TARGET AS LIVE)
CDD=P(SHOOTER CLASS DEAD TARGET AS DEAD)
MEAN NUMBER OF BLUE SHOTS PER TARGET
0
1
2
3
4
5
0
0.2
0.4
0.6
0.8
1.0
CDD

Figure 1.4

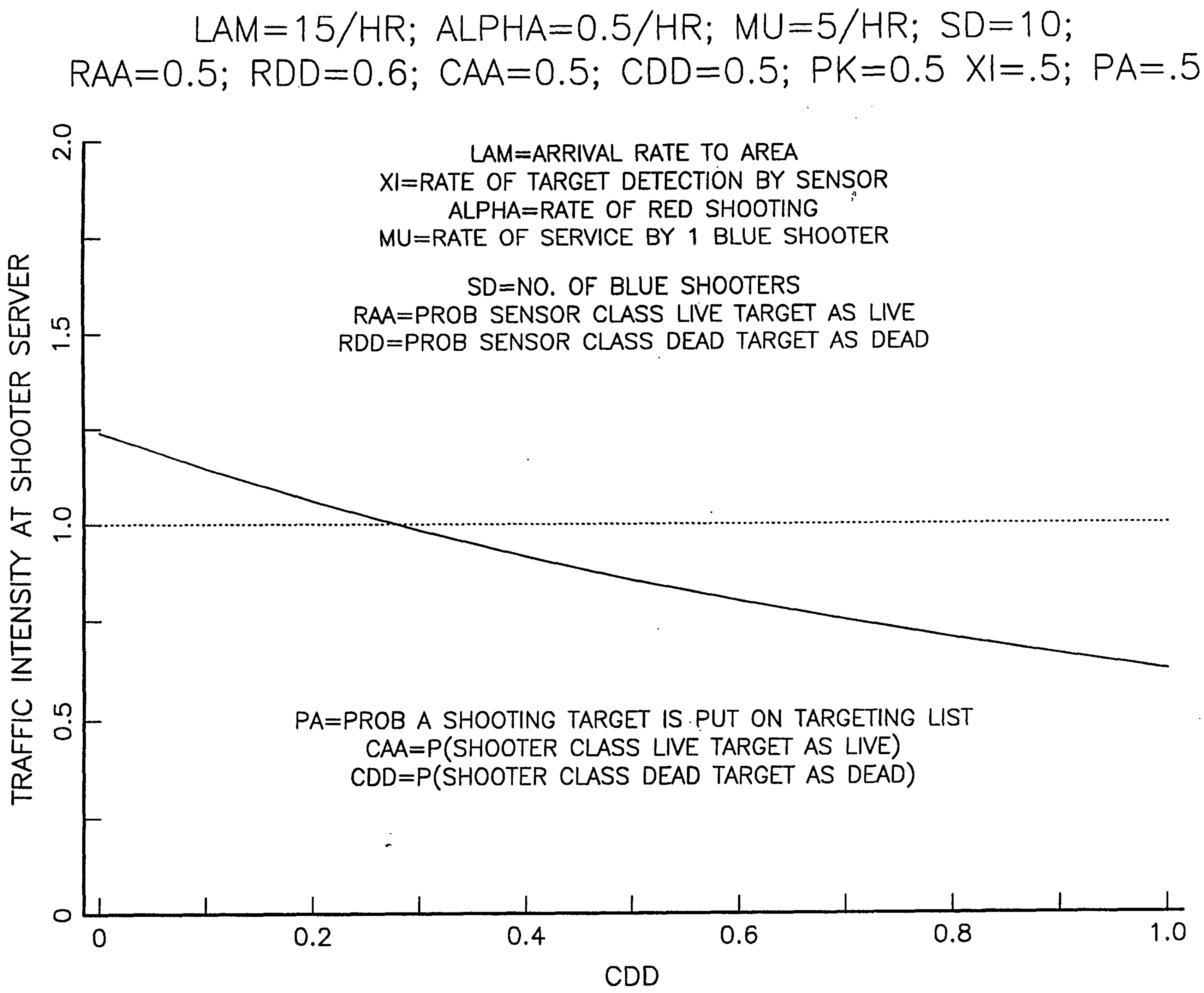

LAM=15/HR; ALPHA=0.5/HR; MU=5/HR; SD=10;
RAA=0.5; RDD=0.6; CAA=0.5; CDD=0.5; PK=0.5 XI=.5; PA=.5
LAM=ARRIVAL RATE TO AREA
XI=RATE OF TARGET DETECTION BY SENSOR
ALPHA=RATE OF RED SHOOTING
MU=RATE OF SERVICE BY 1 BLUE SHOOTER
SD=NO. OF BLUE SHOOTERS
RAA=PROB SENSOR CLASS LIVE TARGET AS LIVE
RDD=PROB SENSOR CLASS DEAD TARGET AS DEAD
PA=PROB A SHOOTING TARGET IS PUT ON TARGETING LIST
CAA=P(SHOOTER CLASS LIVE TARGET AS LIVE)
CDD=P(SHOOTER CLASS DEAD TARGET AS DEAD)
TRAFFIC INTENSITY AT SHOOTER SERVER
0
0.5
1.0
1.5
2.0
0
0.2
0.4
0.6
0.8
1.0
CDD

Figure 1.5

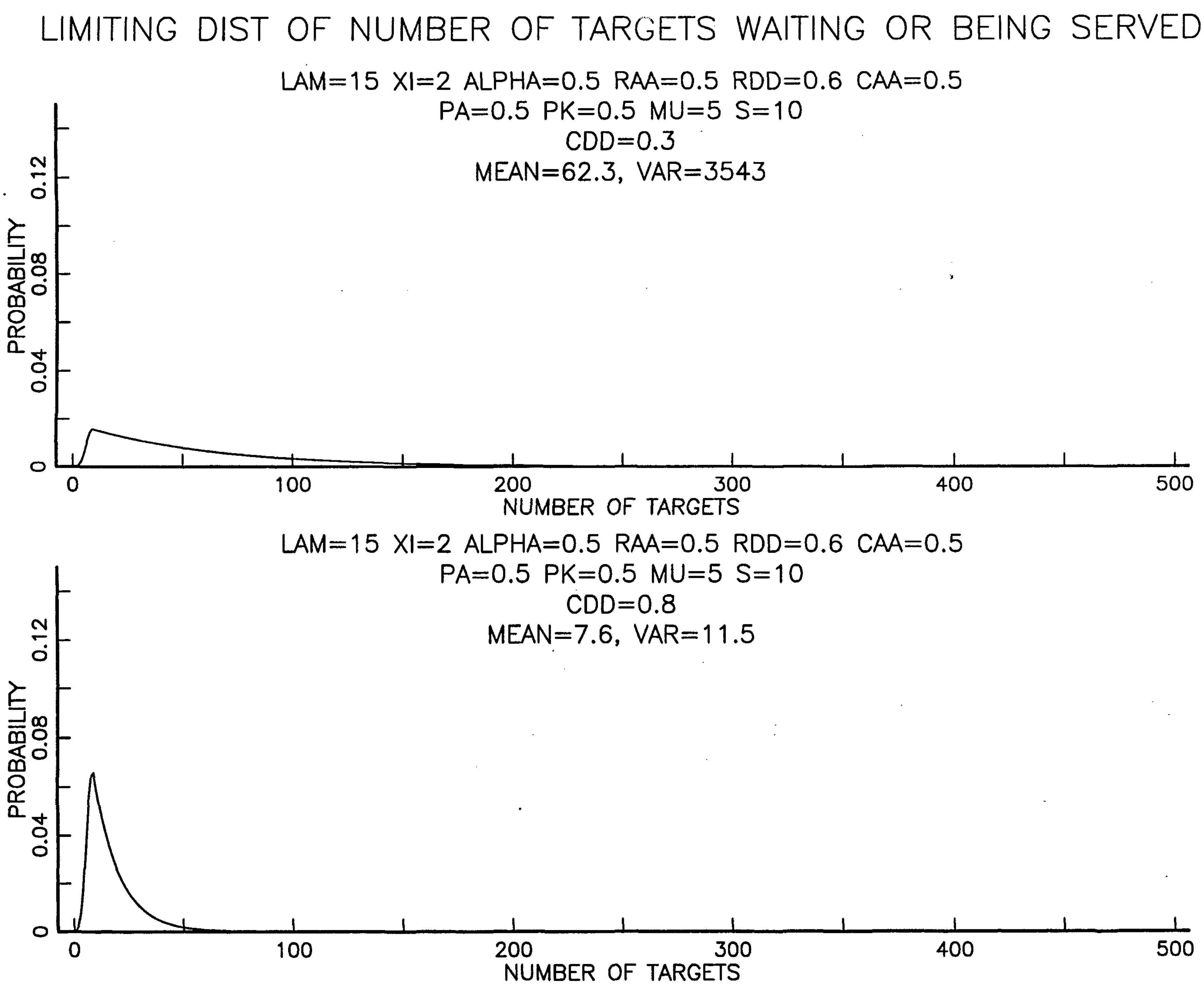

LIMITING DIST OF NUMBER OF TARGETS WAITING OR BEING SERVED
LAM=15 XI=2 ALPHA=0.5 RAA=0.5 RDD=0.6 CAA=0.5
PA=0.5 PK=0.5 MU=5 S=10
CDD=0.3
MEAN=62.3, VAR=3543
PROBABILITY
0
0.04
0.08
0.12
0
100
200
300
400
500
NUMBER OF TARGETS
LAM=15 XI=2 ALPHA=0.5 RAA=0.5 RDD=0.6 CAA=0.5
PA=0.5 PK=0.5 MU=5 S=10
CDD=0.8
MEAN=7.6, VAR=11.5
PROBABILITY
0
0.04
0.08
0.12
0
100
200
300
400
500
NUMBER OF TARGETS

Figure 1.6

with a mean of 62.3. In comparison, when $C_{dd} = 0.8$, the tail of the limiting distribution is much shorter; this shorter tail is reflected in a variance of 11.5 compared to a mean of 7.6.

2. A Fluid Approximation for the Number of Customers Waiting or Being Served in an M/M/s Queue

Consider an M/M/s queue with Poisson arrivals having rate λ, independent exponential service times with mean $1/\mu$ and s servers. Let $N(t)$ be the number of customers waiting or being served at time t. Assume $\lambda < s\mu$.

A deterministic approximation to $\{N(t), t \geq 0\}$ is

$$\frac{dN(t)}{dt} = \lambda - \mu N(t)H(t) \tag{2.1}$$

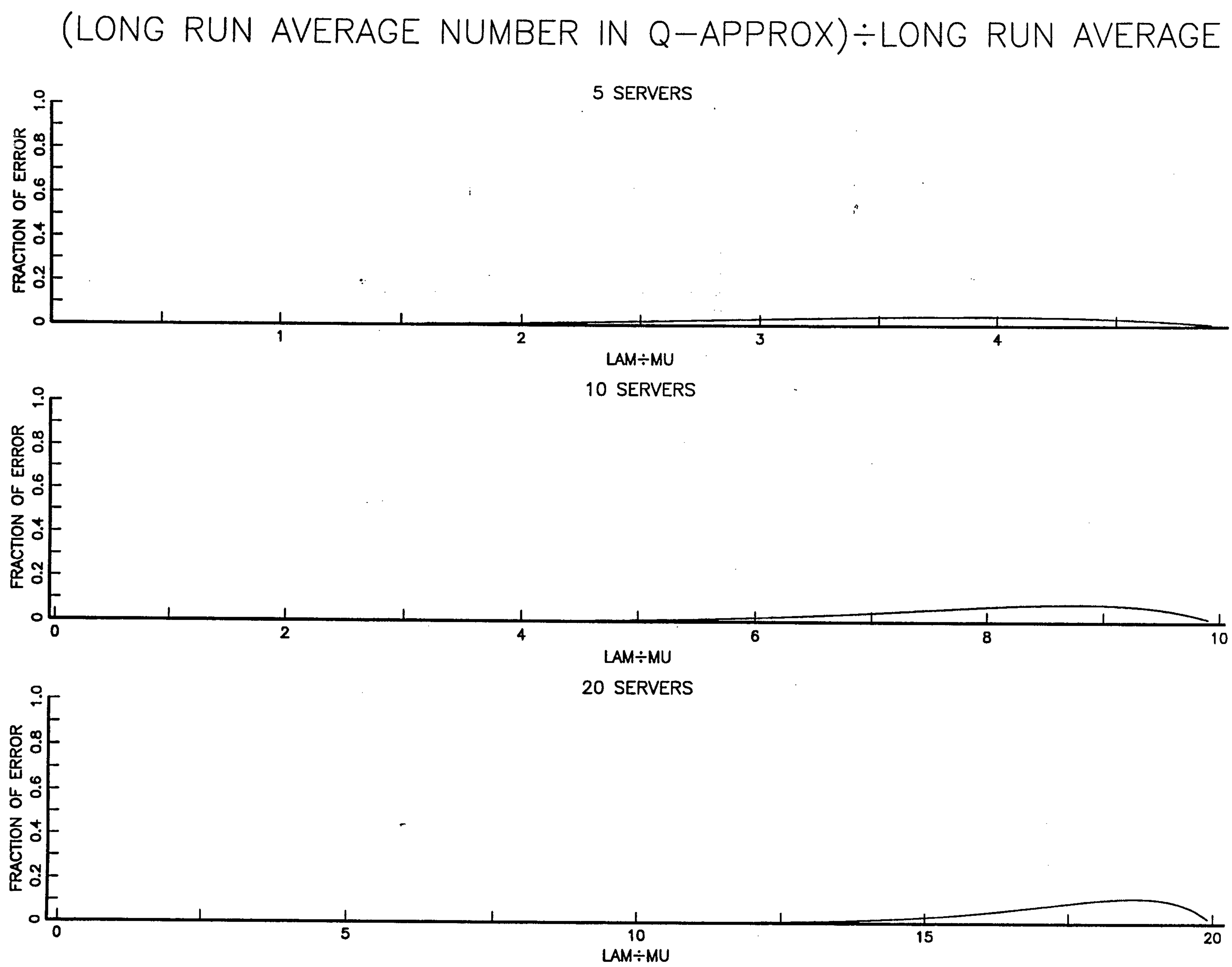
where

$$H(t) = \left[1 + \left(\frac{\mu}{\lambda}\right)\left(\frac{\lambda}{s\mu}\right)^s N(t)\right]^{-1}. \tag{2.2}$$

Letting $t \to \infty$ in (2.1) results in

$$L_a = N(\infty) = \frac{\frac{\lambda}{\mu}}{1 - \left(\frac{\lambda}{s\mu}\right)^s}. \tag{2.3}$$

If $s = 1, 2$, then $N(\infty)$ is exactly equal to L, the long run average number of customers waiting or being served in a M/M/s queue. Figure 2.1 presents plots of $(L - L_a)/L$ for the number of servers $s = 5, 10, 20$ as a function of λ/μ. Note that the approximation L_a is always less than L. Further the approximation becomes less exact as the queues' traffic intensity increases. The size of the error also increases as the number of servers increases. For 10 servers the approximation is at no more than 10% lower than the true. The approximation appears adequate for moderate numbers of servers.



(LONG RUN AVERAGE NUMBER IN Q–APPROX)÷LONG RUN AVERAGE
5 SERVERS
FRACTION OF ERROR
0
0.2
0.4
0.6
0.8
1.0
1
2
3
4
LAM÷MU
10 SERVERS
FRACTION OF ERROR
0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10
LAM÷MU
20 SERVERS
FRACTION OF ERROR
0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
LAM÷MU

Figure 2.1

The approximation with $H(t)$ equal to (2.6) for $s = 1$ has been proposed by Agnew (1976) and Rider (1976); see also Filipiak (1988).

3. A Deterministic Model for Defensive Targeting When Service Success is Unknown and Shooting Strategy is Shoot-Look-Shoot

In this section we present a deterministic or expected-value approximation to the stochastic queuing network model of Section 1. While this deterministic model supplies useful information about systematic process behavior (e.g. time dependencies) it cannot reveal the form of the random variations in targets queued for shooting, as in Table 1.1 and Figure 1.6. (Note that in Table 1.1 the variance of queue length is approximately (mean queue length)^2 for large queue length, descending to a variance of queue length nearly equal to the mean queue length when the latter is small.)

Again suppose attackers that are targets for a defensive force appear in region $\mathfrak{R}$ at a rate λ. The time until an unacquired target that is not itself firing is detected by a surveillance system is distributed exponentially with mean $1/\xi$. A live target that is detected is classified as live and put on the shooter servers' targeting list with probability R_{aa}; with probability $(1 - R_{aa})$ it is misclassified as dead and returns to the unacquired state. A dead target that has not yet been classified as dead is classified as dead when it is acquired with probability R_{dd} and is removed from the system; with probability $(1 - R_{dd})$ it is classified as live, and is erroneously put on the targeting list.

The times between shots by a live target are independent identically distributed exponential (Markovian) with mean $1/\alpha$. An unacquired firing target is detected and put on the shooter's targeting list with probability p_A.

A detected target that has been classified as targetable (perhaps inappropriately because dead) is viewed as queued and awaiting attention of one of s ($s = 1, 2, \ldots$) shooters/"servers". Service time for a shooter can be viewed as a random variable that includes, implicitly, time for target presence in the detected queue, conveyed by C4ISR, to be converted to tracking-firing information; it also includes time of flight in this model.

The shooter-server uses a shoot-look-shoot protocol. Parameters are the same as before: a shot kills a target with probability p_K; BDA occurs immediately after the first shot, so the first shot kills the target then with probability C_{dd} the target is classified as dead and is ignored from then on; with probability $(1 - C_{dd})$ the target is classified as live and it is shot at again. If the first shot misses the target, then with the probability C_{aa} the target is classified as live and the target is shot at a second time; with probability $(1 - C_{aa})$ the target is misclassified as dead and returns to an unacquired state. No battle damage assessment occurs after the second shot; the shooter immediately moves to the next enqueued targetable unit. Once a dead target is classified as dead it is taken out of the system.

3.1 The Effective Arrival Rate of Targets to the Shooter-Server

Let $\lambda_U(A)$, (respectively $\lambda_U(D)$), be the effective arrival rate of live (respectively dead) targets to the undetected state. Let $\lambda_0(A)$, (respectively $\lambda_0(D)$), be the effective arrival rate of live (respectively dead) targets to the shooter-server targeting list for a first shot. Let $\lambda_1(A)$, (respectively $\lambda_1(D)$), be the effective arrival rate of live (respectively dead) targets put again on the targeting list for a second shot.

The effective arrival rates satisfy the following equations.

$$\begin{aligned}\lambda_U(A) &= \lambda + \lambda_0(A)(1-p_K)(1-C_{aa}) + \lambda_1(A)(1-p_K) \\ &\quad + \lambda_U(A)\left[\frac{\xi}{\xi+\alpha}(1-R_{aa}) + \frac{\alpha}{\xi+\alpha}(1-p_A)\right]\end{aligned} \tag{3.1.1a}$$

$$\lambda_0(A) = \lambda_U(A)\left[\frac{\xi}{\xi+\alpha}R_{aa} + \frac{\alpha}{\xi+\alpha}p_A\right] \tag{3.1.1b}$$

$$\lambda_1(A) = \lambda_0(A)(1-p_K)C_{aa} \tag{3.1.1c}$$

$$\lambda_U(D) = \lambda_1(A)p_K + \lambda_1(D) \tag{3.1.1d}$$

$$\lambda_0(D) = \lambda_U(D)[1-R_{dd}] \tag{3.1.1e}$$

$$\begin{aligned}\lambda_1(D) &= \lambda_0(D)[1-C_{dd}] + \lambda_0(A)p_K[1-C_{dd}] \\ &= \lambda_U(D)[1-R_{dd}][1-C_{dd}] + \lambda_0(A)p_K[1-C_{dd}]\end{aligned} \tag{3.1.1f}$$

Note that

$$\lambda_U(D) = \lambda_1(A)p_K + \lambda_U(D)[1-R_{dd}][1-C_{dd}] + \lambda_0(A)p_K[1-C_{dd}]. \tag{3.1.2}$$

Thus,

$$\lambda_U(D) = \frac{\lambda_1(A)p_K + \lambda_0(A)p_K[1-C_{dd}]}{1-[1-R_{dd}][1-C_{dd}]}. \tag{3.1.3}$$

Let

$$\beta = \frac{\xi}{\xi+\alpha}R_{aa} + \frac{\alpha}{\xi+\alpha}p_A; \tag{3.1.4}$$

then,

$$\begin{aligned}\lambda_U(A) &= \frac{\lambda}{\beta\left[1-(1-p_K)(1-C_{aa})-(1-p_K)^2C_{aa}\right]}; \\ \lambda_0(A) &= \frac{\lambda}{1-(1-p_K)(1-C_{aa})-(1-p_K)^2C_{aa}};\end{aligned} \tag{3.1.5}$$

and

$$\lambda_1(A) = \frac{\lambda(1-p_K)C_{aa}}{1-(1-p_K)(1-C_{aa})-(1-p_K)^2C_{aa}}. \tag{3.1.6}$$

Put

$$\lambda_E = \lambda_0(A) + \lambda_1(A) + \lambda_0(D) + \lambda_1(D). \tag{3.1.7}$$

3.2 A Deterministic Network Queuing Model Involving Shoot-Look-Shoot

Consider the following variables.

$A_U(t)$ = number of undetected live targets at time t

$A_0(t)$ = number of detected live targets that are on the shooter servers' targeting list and are waiting for the first shot at time t

$A_1(t)$ = number of detected live targets waiting for the second shot at time t

$D_U(t)$ = number of undetected dead targets that have not yet been classified as dead at time t

$D_0(t)$ = number of detected dead targets that have not yet been classified as dead and are waiting for the first shot

$D_1(t)$ = number of detected dead targets that have not yet been classified as dead and are waiting for the second shot

$K_a(t)$ = Number of Reds killed by time t

$K(t)$ = Number of Reds killed by time t which are classified as dead

$R(t)$ = Number of Red shots by time t

$B(t)$ = Number of Blue shots by time t

The variable $X_S(t)$ in the stochastic model of Section 1 corresponds to $A_0(t) + A_1(t) + D_0(t) + D_1(t)$.

Consider the following parameters.

λ = Rate of arrival of Red attackers to region

μ = Rate at which acquired targets are served by a shooter-server

ν = Rate at which acquired live targets are lost from track

α = Rate at which attackers are active

ξ = Rate at which a target is detected by the defender sensors

p_K = Probability a live Red target is killed

C_{aa} = Probability shooter classifies a live target as live after shooting

C_{dd} = Probability shooter classifies a dead target as dead after shooting

R_{aa} = Probability a live target is classified as live by a defender sensor

R_{dd} = Probability a dead target is classified as dead by a defender sensor

p_A = Probability an active shooting Red is acquired by the server

Let

$$H(t) = \left[1 + \frac{\mu}{\lambda_E}\left(\frac{\lambda_E}{s\mu}\right)^s [A_0(t) + A_1(t) + D_0(t) + D_1(t)]\right]^{-1}; \qquad (3.2.1)$$

$H(t)$ is a term to approximate the behavior of an M/M/s queue (as described in Section 2).

Consider the following deterministic model as an approximation to the network of queues model (as described in Section 1).

$$\frac{dA_U(t)}{dt} = \underbrace{\lambda}_{\substack{\text{arrival rate}\\\text{of targets}\\\text{to area}}} + \underbrace{\nu(A_0(t)+A_1(t))}_{\substack{\text{rate of loss of active}\\\text{Reds from track}}} - \underbrace{\alpha p_A A_U(t)}_{\substack{\text{rate of}\\\text{acquisition}\\\text{due to Red}\\\text{activity}}} - \underbrace{\xi R_{aa} A_U(t)}_{\substack{\text{rate of acquisition}\\\text{due to sensors}}} \qquad (3.2.2a)$$

$$+ \underbrace{\mu(1-p_K)A_1(t)H(t)}_{\text{active still alive after 2 shots}} + \underbrace{\mu(1-p_K)A_0(t)(1-C_{aa})H(t)}_{\substack{\text{active Red alive after first shot}\\\text{misclassified as dead}}}$$

$$\frac{dA_0(t)}{dt} = \alpha p_A A_U(t) + \xi R_{aa} A_U(t) - \mu A_0(t)H(t) - \nu A_0(t) \qquad (3.2.2b)$$

$$\frac{dA_1(t)}{dt} = \underbrace{\mu A_0(t)(1-p_K)C_{aa}H(t)}_{\substack{\text{active Red alive after first shot,}\\\text{classified as alive}}} - \underbrace{\mu A_1(t)H(t)}_{\substack{\text{rate at which Red}\\\text{actives are shot at}\\\text{a second time}}} - \nu A_1(t) \qquad (3.2.2c)$$

$$\frac{dD_U(t)}{dt} = \underbrace{\mu A_1(t) p_K H(t)}_{\substack{\text{active Red killed on} \\ 2^{\text{nd}} \text{ shot}}} - \underbrace{\xi D_U(t)}_{\substack{\text{rate at which} \\ \text{dead Red not} \\ \text{yet classified as} \\ \text{dead is acquired} \\ \text{by sensor}}} + \underbrace{\mu D_1(t) H(t)}_{\substack{\text{rate at which} \\ \text{dead Red not yet} \\ \text{classified as dead} \\ \text{is shot at second time}}} \tag{3.2.2d}$$

$$\frac{dD_0(t)}{dt} = \underbrace{\xi(1 - R_{dd}) D_U(t)}_{\substack{\text{rate at which} \\ \text{dead Red not yet} \\ \text{classified as dead} \\ \text{is acquired by sensor} \\ \text{and classified as live}}} - \underbrace{\mu D_0(t) H(t)}_{\substack{\text{rate at which} \\ \text{dead targets not} \\ \text{classified as dead} \\ \text{are shot at}}} \tag{3.2.2e}$$

$$\frac{dD_1(t)}{dt} = \underbrace{\mu p_K (1 - C_{dd}) A_0(t) H(t)}_{\substack{\text{rate at which a live Red is} \\ \text{killed on } 1^{\text{st}} \text{ shot but is} \\ \text{misclassified as live}}} + \mu D_0(t)(1 - C_{dd}) H(t) - \mu D_1(t) H(t) \tag{3.2.2f}$$

$$\frac{dK_a(t)}{dt} = \mu p_K \left(A_0(t) + A_1(t)\right) H(t) \tag{3.2.2g}$$

$$\frac{dK(t)}{dt} = \mu p_K C_{dd} A_0(t) H(t) + \xi D_U(t) R_{dd} + \mu C_{dd} D_0(t) H(t) \tag{3.2.2h}$$

$$\frac{dR(t)}{dt} = \alpha\left(A_U(t) + A_0(t) + A_1(t)\right) \tag{3.2.2i}$$

$$\frac{dB(t)}{dt} = \mu\left(A_0(t) + A_1(t) + D_0(t) + D_1(t)\right) H(t) \tag{3.2.2j}$$

3.3 Numerical Results

Consider a model with the following parameters: $\lambda = 15$, $\xi = 2$, $\alpha = 0.5$, $R_{aa} = 0.5$, $R_{dd} = 0.6$, $C_{aa} = 0.5$, $p_A = 0.5$, $p_K = 0.5$, $\mu = 5$, $s = 10$. Table 3.1 displays the long run average number of targets waiting or being served by the shooter-servers and the long run average number of live targets waiting or being served by the shooter-servers as a function of C_{dd} for the queuing network model of Section 1. Also displayed are the values of the total number of targets waiting or being served at the shooter servers, $A_0(300) + A_1(300) + D_0(300) + D_1(300)$, as a function of C_{dd} and the long run average number of live targets waiting or being served at time 300, $A_0(300) + A_1(300)$, for the deterministic model. The deterministic model was evaluated using the $4^{th}/5^{th}$ order Runge-Kutta-Fehlberg method as implemented in MATLAB. The agreement is good where both models apply. The deterministic model is able to (quickly) estimate the expected number of live (opponent) targets at time t (= 300, here) even when the sensor-shooter system is saturated.

Table 3.1
Targets Waiting or Being Served by the Shooter Servers

C_{dd}	M/M/10 Average Number of Targets	Deterministic Number of Targets at time 300	M/M/10 Average Number of Live Targets	Deterministic Number of Live Targets at time 300
0.1	—	3687.0	—	2797.0
0.2	—	2279.0	—	1524.0
0.3	65.3	63.6	39.8	38.9
0.4	16.9	15.7	11.1	10.3
0.5	11.8	10.8	8.2	7.6
0.6	9.6	9.0	7.2	6.7
0.7	8.4	7.9	6.7	6.4
0.8	7.6	7.3	6.5	6.2
0.9	7.0	6.7	6.3	6.1
1.0	6.4	6.3	6.2	6.1

— queuing model is saturated

4. A Nonstationary Network Queuing Model Involving Shoot-Look-Shoot

Letting $\lambda \to 0$ in the model of Subsections 3.1 and 3.2 will result in $\lambda_E = 0$. The function H of 3.2.1 will tend to 1 and the service process will be similar to an infinite server queue.

Since it is important to model the transient behavior of the system under a nonstationary arrival process of targets, we will modify the effective arrival rates as follows:

$$\lambda_0(A,t) = A_U(t)[\xi R_{aa} + \alpha p_A] \tag{4.1a}$$

$$\lambda_1(A,t) = \lambda_0(A,t)(1-p_K)C_{aa} \tag{4.1b}$$

$$\lambda_0(D,t) = D_U(t)\xi[1-R_{dd}] \tag{4.1c}$$

$$\lambda_1(D,t) = \lambda_0(D,t)[1-C_{dd}] + \lambda_0(A,t)p_K[1-C_{dd}]. \tag{4.1d}$$

The effective arrival rate at the shooter server is

$$\lambda_E(t) = \lambda_0(A,t) + \lambda_1(A,t) + \lambda_0(D,t) + \lambda_1(D,t). \tag{4.2}$$

Put

$$H(t) = \left[1 + \frac{\mu}{\lambda_E(t)}\left(\frac{\lambda_E(t)}{s\mu}\right)^s [A_0(t) + A_1(t) + D_0(t) + D_1(t)]\right]^{-1}. \tag{4.3}$$

$H(t)$ is a term to approximate the behavior of the M/M/s queue.

The deterministic model equations of Section 3 remain the same except for replacing λ by (possibly) $\lambda(t)$ and using $H(t)$ of (4.3).

4.1 Numerical Results

Consider a model with the following parameters: $\lambda = 15$, $\xi = 2$, $\alpha = 0.5$, $R_{aa} = 0.5$, $R_{dd} = 0.6$, $C_{aa} = 0.5$, $p_A = 0.5$, $p_K = 0.5$, $\mu = 5$, $s = 10$. Table 4.1 displays the long run average number of targets waiting or being served by the shooter-servers and the long run average number of live targets waiting or being served by the shooter-servers as a function of C_{dd} for the queuing network model of Section 1. Also displayed are the values of the total number of targets waiting or being served at the shooter servers, $A_0(100) + A_1(100) + D_0(100) + D_1(100)$, as a function of C_{dd} and the long run average number of live targets waiting or being served at time 100, $A_0(100) + A_1(100)$, for the deterministic model of Subsection 4.2. The deterministic model was evaluated using the 4th/5th order Runge-Kutta-Fehlberg method as implemented in MATLAB. Comparison with Table 3.1 indicates that the deterministic model with effective arrival rate (4.1a) – (4.1d) and (4.2) gives the same steady state results as the deterministic model of Section 3 for most cases. The effective arrival rate (4.1a) – (4.1d) and (4.2) is preferable since it will allow the deterministic model to gracefully decrease if the arrival rate of targets into the area at time t, $\lambda(t)$ tends to 0.

Table 4.1
Targets Waiting or Being Served by the Shooter Servers

C_{dd}	M/M/10 Average Number of Targets	Deterministic Number of Targets at time 300	M/M/10 Average Number of Live Targets	Deterministic Number of Live Targets at time 300
0.1	—	1163.0	—	662.6
0.2	—	572.3	—	335.2
0.3	65.3	61.8	39.8	37.7
0.4	16.9	15.7	11.1	10.3
0.5	11.8	10.8	8.2	7.6
0.6	9.6	9.0	7.2	6.7
0.7	8.4	7.9	6.7	6.4
0.8	7.6	7.3	6.5	6.2
0.9	7.0	6.7	6.3	6.1
1.0	6.4	6.3	6.2	6.1

— queuing model is saturated

5. Summary

The present paper finds the explicit long-run stochastic behavior for a scenario that envisions targets (Red assets) entering a region, being detected and targeted. The surveillance rate, probability of correct classification, kill probability, and BDA capabilities are all bounded, so targeting is conducted in a realistic environment of imperfect and uncertain sensor-shooter system performances. Such models permit quick investigation of tradeoffs in system element capabilities. The explicit stochastic representation provides insights into the ultimate variabilities and uncertainties encountered when detection, classification, and BDA are collectively or individually mediocre to poor. Such conditions can be induced by effects that are not explicitly modeled here, such as Red use of low-value decoys and sophisticated "play dead" tactics by live assets that have received plausible (Blue) fire.

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